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VIBRATION OF TWO CIRCULAR STAMPS IN A LAYERED MEDIUM

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The problem of the harmonic oscillations on the surface of an elastic layered medium by two circular stamps of radii a_1 and a_2 is considered. The spacing between the centers of the stamps is $b > a_1 + a_2$. It is assumed that there is no friction in the area of contact.

By using a method developed in [1], the initial system is reduced to a system of Fredholm equations of the second kind, for whose solution approximate methods are proposed. On the basis of the results obtained, an applied theory for the vibrations of two stamps can be constructed which also takes account of the dispersion properties of the medium, in contrast to all other known applied theories.

It is simple to investigate the case of vibrations of a system of n stamps by the method elucidated in this paper, however, we limit ourselves to the case of two stamps for the sake of brevity.

1. Letting Ω_1 and Ω_2, respectively, denote the domains occupied by stamps of radii a_1 and a_2, we represent the integral equations of the problem as

$$\iint_{\Omega_1} k(r, \rho, \varphi, \psi)\, q_1(\rho, \psi)\, \rho d\rho\, d\psi + \iint_{\Omega_2} k(r, \rho, \varphi, \psi)\, q_2(\rho, \psi)\, \rho d\rho\, d\psi = f_k(r, \varphi) \tag{1.1}$$

$$r, \varphi \in \Omega_k, \quad k = 1, 2$$

$$k(r, \rho, \varphi, \psi) = \int_\Gamma K(u)\, J_0(uR)\, u du, \quad R = \sqrt{r^2 + \rho^2 - 2r\rho\cos(\varphi - \psi)}$$

Here $\operatorname{Re} q_k e^{-i\omega t}$ are the contact stresses under the stamps vibrating according to the law $\cos \omega t$, and $\operatorname{Re} f_k e^{-i\omega t}$ are the displacements of points at the bottom of the stamps, where $k = 1$ ($k = 2$) in the case of the stamp of radius a_1 (of radius a_2).

If the medium is a system of layers lying on an elastic half-space, then the function $K(u)$ is an even analytic function in the complex plane which has single real poles, where the positive ones will be denoted by ζ_k ($k = 1, 2, \ldots, n$) and the branch points $\pm A_1$ and $\pm A_2$. The branch points are connected to infinity by slits in the first and third quadrants. The conditions of wave radiation in the medium reduce to the requirement that the contour Γ be in the fourth quadrant with origin at zero and terminus at the point $\infty - i0$. Let us note that the function $K(u)$ depends on all the geometric and mechanical characteristics of the layers and the half-space as well as on the frequency ω of stamp vibration.

The following is valid

Theorem 1 (uniqueness). Let the function $K(u)$ be real on a segment containing the poles ζ_k from which the residues are of the same sign; let the imaginary part of the function $K(u)$ have the same sign on some segment of the real axis, and not change to its opposite. Then (1.1) cannot have more than one solution in the class L_p $(p > 1)$.

The method of proving the theorem is an incidental modification of that elucidated in [2] and is not presented here for the sake of brevity.

2. Let us expand the right sides f_k and the unknown functions q_k in Fourier series of the form

$$f_k(r, \varphi) = \sum_{l=-\infty}^{\infty} f_{kl}(r) e^{il\varphi} \tag{2.1}$$

$$q_k(\rho, \psi) = \sum_{p=-\infty}^{\infty} q_{kp}(\rho) e^{ip\psi}$$

Consequently, by applying addition formulas for cylinder functions, we represented (1.1) as a system of the form

$$\int_0^{a_1}\int_\Gamma K(u)\, u\, J_p(ur) J_p(u\rho) q_{1p}(\rho)\rho du\, d\rho + \int_0^{a_2}\int_\Gamma K(u)\, u \sum_{s=-\infty}^{\infty} (-1)^{s+p} \times \tag{2.2}$$
$$J_p(ur) J_{s-p}(ub) J_s(u\beta)\, du\, d\beta = f_{1p}(r), \quad 0 \leqslant r \leqslant a_1, \quad p = 0, \pm 1, \pm 2, \ldots$$

$$\int_0^{a_2}\int_\Gamma K(u)\, u\, J_\tau(ur) J_\tau(u\rho) q_{2\tau}(\rho)\rho du\, d\rho + \int_0^{a_1}\int_\Gamma K(u)\, u \sum_{m=-\infty}^{\infty} (-1)^{m+\tau} \times \tag{2.3}$$
$$J_\tau(ur) J_{\tau-m}(ub) J_m(u\alpha) q_{1m}(\alpha)\alpha du\, d\alpha = f_{2\tau}(r), \quad 0 \leqslant r \leqslant a_2$$
$$\tau = 0, \pm 1, \pm 2, \ldots$$

Let us introduce the function $\varkappa_n(u, a_k, s)$, defined by regularity conditions in the lower half-plane, by the absence of zeros there, and by the behavior

$$\begin{aligned} &i\sqrt{a}\varkappa_1(u, a, s) H_s^{(2)}(ua) \to 1 \\ &\sqrt{a}\varkappa_2(u, a, s) J_s(ua) \to 1 \end{aligned} \qquad \operatorname{Im} u \to -\infty \tag{2.4}$$

Lemma. The solution $q_{\kappa p}(r)$ of the system (2.1) have representations of the form

$$q_{kp}(r) = \int_0^\infty J_p(\eta r) K^{-1}(\eta) F_{kp}(\eta)\eta d\eta + \int_{\Gamma_1} J_p(ur) K^{-1}(u) Z(k, p, u)\, du \tag{2.5}$$

Here $Z(k, p, u)$ are regular functions decreasing in the lower half-plane.

In addition, the representation

$$f_{kp}(r) = \int_0^\infty F_{kp}(\eta)\eta J_p(\eta r)\, d\eta \tag{2.6}$$

is taken.

The function $F_{kp}(\eta)$ is selected in such a way that it vanishes at real zeros of the function $K(\eta)$.

We show that such a representation is always easily constructed.

Let z_m, $m = 1, 2, \ldots, n$ denote real zeros of the function $K(\eta)$. We construct the following continuation $f_{kp}^*(r)$ of the function $f_{kp}(r)$

$$f_{kp}^*(r) = f_{kp}(r), \quad 0 < r < a \quad a \equiv a_\kappa$$
$$f_{kp}^*(r) = c_{kpm} r^{-p}, \quad am \leqslant r \leqslant a(m+1)$$
$$f_{kp}^*(r) \equiv 0, \quad r > an$$

The Bessel transformation of the function $f_{kp}{}^*$ will indeed be one of the values of $F_{kp}(\eta)$ if the constants c_{kpm} are selected from the condition $F_{kp}(z_m) = 0$.

It is hence clear that a nondenumerable set of representations (2.6) can be constructed; the continuation $f_{kp}{}^*(r)$ with continuous derivatives of the order needed must be constructed to obtain the rapidly decreasing functions $F_{kp}(\eta)$.

The derivation of (2.5) is based on the continuation of the right sides of (2.2) into the domain $r > a_1$ and $r > a_2$, respectively, with subsequent solution of the system, considered on a half-axis, by using the Bessel transform.

Now, let us insert (2.5) into the system (2.2) and let us integrate. Then by using known addition formulas for Bessel functions we arrive at the following system to determine the unknowns Z_{ksu}:

$$\int_{\Gamma_1} \frac{Q(\alpha, u, a_1, p)}{(\alpha^2 - u^2) K(u)} Z(1, p, u)\, du = \tag{2.7}$$

$$\int_{\Gamma_1} \sum_{s=-\infty}^{\infty} (-1)^{s+p} \frac{E(\alpha, u, a_2, p, s)}{(\alpha^2 - u^2) K(u)} Z(2, s, u)\, du + D(\alpha, a_1)$$

$$\int_{\Gamma_1} \frac{Q(\alpha, u, a_2, p)}{(\alpha^2 - u^2) K(u)} Z(2, s, u)\, du =$$

$$\int_{\Gamma_1} \sum_{s=-\infty}^{\infty} (-1)^{s+p} \frac{E(\alpha, u, a_1, p, s)}{(\alpha^2 - u^2) K(u)} Z(1, s, u)\, du +$$

$$D(\alpha, a_2), \quad p = 0, \pm 1, \pm 2$$

Here

$$Q(\alpha, u, a, p) = a\alpha H_{p+1}^{(2)}(\alpha a) J_p(ua) - auH_p^{(2)}(\alpha a) J_{p+1}(ua)$$

$$E(\alpha, u, a, p, s) = [-a\alpha J_{s+1}(\alpha a) J_s(ua) + auJ_s(\alpha a) J_{s+1}(ua)] H_{s-p}^{(2)}(\alpha b)$$

$$D(\alpha, a_k) = a_k \int_0^{\infty} \frac{\alpha H_{p+1}^{(2)}(\alpha a_k) J_p(\eta a_k) - \eta H_p^{(2)}(\alpha a_k) J_{p+1}(\eta a_k)}{(\eta^2 - \alpha^2) K(\eta)} F_{kp}(\eta)\, \eta\, d\eta$$

The contour Γ_1 is located near the lower boundary of the domain S in which the function $K(u)$ is regular. Having determined the unknowns $Z(k, p, u)$ from the system (2.7), we find the solution of the problem.

3. To reduce the system (2.7) to equations of the second kind, let us study the properties of the solution of an equation of the form

$$\frac{1}{4\pi^2} \int_{\Gamma_2} \int_{\Gamma_1} \frac{K_+(\alpha) X(u)\, du d\alpha}{(\alpha - z)(u - \alpha) K_+(u)} = -Y(z) \tag{3.1}$$

Here the contour Γ_2 is located in the domain of regularity of the function $K(u)$ above the contour Γ_1, while z is located above the contour Γ_2. The function $X(z)$ is regular in the domain S and decreases according to a power law there. As is easy to see, the properties of the function $K(u)$ assure a behavior in the regularity domain, described by the estimate $cu^{-1/2}[1 + o(1)] \quad |u| \to \infty$

for $K_+(u)$ obtained as a result of factorization of $K(u)$ relative to the contour Γ_1, where $K_+(u)$ is regular above the contour Γ_1

The function $Y(z)$ is regular in the domain S and the upper half-plane, decreases according to a power-law there, and admits of analytic continuation in the lower half-plane.

Continuing the left side of (3. 1) analytically in the lower half-plane, we have

$$\frac{1}{4\pi^2}\int\limits_{\Gamma_3}\int\limits_{\Gamma_1}\frac{K_+(\alpha)X(u)\,du\,d\alpha}{(\alpha-z)(u-\alpha)K_+(u)}+\frac{i}{2\pi}K_+(z)\int\limits_{\Gamma_1}\frac{X(u)\,du}{(u-z)K_+(u)}=-Y \qquad (3.2)$$

Here the contours Γ_1, Γ_2 are located in the domain S, and Γ_3 is above Γ_1, where z is between them. The first integral written down is evidently zero; this follows from the fact that the integral with respect to α is zero since the integrand is regular above the contour Γ_3.

Continuing the function represented by the second integral into the lower half-plane and introducing the notation (z is below the contour Γ_3)

$$\int\limits_{\Gamma_3}\frac{X(u)\,du}{(u-z)K_+(u)}=\frac{i}{2\pi}R_-(z)$$

we have

$$X(z)=Y(z)+K_+(z)R_-(z) \qquad (3.3)$$

The relationship (3. 3) yields a general representation of the solution of (3. 1). By using this result we complete the transformation of the system (2. 7). We perform the following factorization with respect to the contour Γ_1

$$K[\mu_k(u)]=K_{k+}(u)K_{k-}(u),\quad \mu_k=\sqrt{u^2+p^2a_k^{-2}} \qquad (3.4)$$

Here the radical is defined on a Riemann plane with a slit connecting the points $\pm ip/a$, and by the condition of positivity for $u>0$. The factorization of the function $K[\mu_k(u)]$ evidently degenerates into the factorization of the function $K(u)$ as $p/a\to 0$. The slit exerts no influence in performance of the factorization because of the regularity and evenness of the function $K(u)$ on the contour Γ_1.

We multiply the first and second relationships in (2. 7), respectively, by

$$\frac{\alpha K_{k+}[\lambda_k(\alpha)]\varkappa_1[\lambda_k(\alpha),a_k,p]}{\lambda_k(\alpha)[\lambda_k(\alpha)-\lambda_k(z)]}\ (\lambda_k(\alpha)=\sqrt{\alpha^2-p^2a_k^{-2}}),\quad k=1,2$$

Here the radical is defined on a Riemann plane with a slit in the upper half-plane connecting the points $u=\pm p/a_k$ and by the condition $\lambda>0$ as $u\to\infty$.

We introduce a change of the unknown by setting

$$Z(k,p,u)=\varkappa_2[\lambda_k(u),a_k,p]X[k,p,\lambda_k(u)]u\lambda_k^{-1}(u)K_{k-}[\lambda_k(u)]$$

Now integrating (2. 7) along the contour Γ_2 lying above Γ_1 in the plane of regularity of $K(u)$ we arrive at relationships for which the first becomes

$$\frac{1}{4\pi^2}\int\limits_{\Gamma_2}\int\limits_{\Gamma_1}\frac{Q(\alpha,u,a_1,p)K_{1+}[\lambda_1(\alpha)]\varkappa_1[\lambda_1(\alpha),a_1,p]\varkappa_2[\lambda_1(u),a_1,p]}{[\lambda_1(\alpha)-\lambda_1(z)](\alpha^2-u^2)K_{1+}[\lambda_1(u)]\lambda_1(u)\lambda_1(\alpha)}\times \qquad (3.5)$$

$$X[1,p,\lambda_1(u)]\,\alpha u\,du\,d\alpha=\frac{1}{4\pi^2}\int\limits_{\Gamma_2}\int\limits_{\Gamma_1}\sum_{s=-\infty}^{\infty}(-1)^{s+p}\times$$

$$\frac{E(\alpha,u,a_2,p,s)\varkappa_1[\lambda_1(\alpha),a_1,p]\varkappa_2[\lambda_2(\alpha),a_2,s]K_{1+}[\lambda_1(\alpha)]}{[\lambda_1(\alpha)-\lambda_1(z)](\alpha_2-u^2)K_{2+}[\lambda_2(u)]\lambda_1(\lambda)\lambda_2(u)}\times$$

$$X[2, s, \lambda_2(u)]\, au\, du\, d\alpha + \frac{1}{4\pi^2}\int\limits_{\Gamma_2} \frac{D(\alpha, a_1) K_{1+}[\lambda_1(\alpha)]\, \varkappa_1[\lambda_1(\alpha), a_1, p]}{[\lambda_1(\alpha) - \lambda_1(z)]\, \lambda_1(\alpha)}\, \alpha d\alpha$$

The point z is here above the contour Γ_2, while the contour Γ_2 is above Γ_1. The second relationship, omitted for the sake of brevity, takes an analogous form.

Using uniform asymptotic estimates of the Bessel functions with large complex arguments and subscripts in the lower half-plane, we obtain the left side of the relationship in the form

$$\frac{1}{4\pi^2}\int\limits_{\Gamma_2}\int\limits_{\Gamma_1} \frac{K_{1+}[\lambda_1(\alpha)]\, u\alpha X[1, p, \lambda_1(u)]\, du\, d\alpha}{[\lambda_1(\alpha) - \lambda_1(z)][\lambda_1(u) - \lambda_1(\alpha)]\, K_{1+}[\lambda_1(u)]\, \lambda_1(u)\, \lambda_1(\alpha)} \qquad (3.6)$$

Passing to a new complex plane by using the substitution $\lambda = \lambda_1(u)$, we obtain the already studied operator (3.1).

With this substitution the contours Γ_1, Γ_2 in the λ-plane are mapped into certain other contours γ_1, γ_2 which also lie in the lower half-plane and retain their behavior at infinity.

We add and subtract the expression on the left in (3.1, considered in the λ-plane, in the left side of the relationship (3.5) in the plane of the complex variable λ. Then, using the representation (3.3) and inserting it into (3.5), we obtain an equation of the form

$$y(1, p, z) = \frac{1}{4\pi^2}\int\limits_{\gamma_2}\int\limits_{\gamma_1} \frac{K_{1+}(\alpha)\, C(\alpha, u, a_1, p)\, y(1, p, u)}{(\alpha - z)(\alpha^2 - u^2)\, K_{1+}(u)}\, du\, d\alpha + \qquad (3.7)$$

$$\frac{1}{4\pi^2}\int\limits_{\gamma_2}\int\limits_{\gamma_1}\sum_{s=-\infty}^{\infty} (-1)^{s+p} \frac{K_{1+}(\alpha)\, D(\alpha, u, a_1, a_2, p, s)\, y(2, s, u)}{(\alpha - z)(\alpha^2 - u^2)\, K_{2+}(u)} \times$$

$$du\, d\alpha + F(1, z, p)$$

The second group of equations becomes

$$y(2, p, z) = \frac{1}{4\pi^2}\int\limits_{\gamma_2}\int\limits_{\gamma_2} \frac{K_{2+}(\alpha)\, C(\alpha, u, a_2, p)\, y(2, p, u)}{(\alpha - z)(\alpha^2 - u^2)\, K_{2+}(u)}\, du\, d\alpha + \qquad (3.8)$$

$$\frac{1}{4\pi^2}\int\limits_{\gamma_2}\int\limits_{\gamma_1}\sum_{s=-\infty}^{\infty} \frac{K_{2+}(\alpha)\, D(\alpha, u, a_2, a_1, p, s)\, y(1, s, u)}{(\alpha - z)(\alpha^2 - u^2)\, K_{1+}(u)}\, du\, d\alpha + F(2, z, p)$$

Here

$$C(\alpha, u, a_1, p) = -\varkappa_1(\alpha, a_1, p)\varkappa_2(u, a_1, p) Q[\mu_1(\alpha), \mu_1(u), a_1, p] + (\alpha + u) \qquad (3.9)$$

$$D(\alpha, u, a_1, a_2, p, s) = -\varkappa_1(\alpha, a_1, p)\varkappa_2(u, a_2, s) E[\mu_1(\alpha), \mu_2(u), a_2, p, s]\, H_{s-p}^{(2)}[\mu_1(\alpha), b],$$

$$F(k, z, p) = \frac{1}{4\pi^2}\int\limits_{\gamma_2} \frac{D[\mu_k(\alpha), a_k]\, K_{k+}(\alpha)\, \varkappa_1(\alpha, a_k, p)}{\alpha - z}\, d\alpha$$

We reduce (3.7) — (3.9) to equations of the second kind with completely continuous operators in the space of functions, continuous with weight u^λ $(0 < \lambda < 1)$ on the contour Γ_1 and which vanish at infinity.

Let us first study the asymptotic behavior of the integrand for large $|u|$, $|\alpha|$, p, s on contours in the lower half-plane. Using the uniform asymptotic expansions of the Bessel functions by setting

$$\varkappa_1(u, a, p) = \frac{\sqrt{a}}{2\pi\sqrt{p^2 - u^2a^2}\, K_p(iua)} \qquad (3.10)$$

$$\varkappa_2(u, a, p) = \frac{2}{\pi} i \sqrt{a}\, K_p(iua)$$

we have

$$C(\alpha, u, a_1, p) = c + O(\alpha^{-1}, u^{-1}, p^{-1})$$

We use the relationship

$$\int_{\Gamma_2}\int_{\Gamma_1} \frac{\Phi(\alpha, u)\, du\, d\alpha}{(\alpha - z)(\alpha^2 - u^2)} = 2\pi^2 \frac{\Phi(z, z)}{z} + 2\pi i \int_{\Gamma_3} \frac{\Phi(z, u)\, du}{z^2 - u^2} + \qquad (3.11)$$

$$\frac{1}{2\pi i} \int_{\Gamma_4}\int_{\Gamma_3} \frac{\Phi(\alpha, u)\, du\, d\alpha}{(\alpha - z)(\alpha^2 - u^2)}$$

Here the contour Γ_k lies above Γ_{k-1}. The point z is between the contours Γ_2 and Γ_3. All the contours lie in the domain S. The function $\Phi(\alpha, u)$ is regular in both variables in this domain, and decreases as $|u|^{-1-\varepsilon}$, $\varepsilon > 0$ in each variable on the contours.

Now taking the integrand of the first terms in the right side of (3.7) as $\Phi(\alpha, u)$ in (3.11), we note that the term outside the integral is proportional to $y(1, p, z)$. Let us transfer it into the left side of (3.7) and combine it with the unknown $y(1, p, z)$ there.

The term outside the integral drops out because of an analogous transformation of the second term (sum) in (3.7).

Solving the transformed equation (3.7) for $y(1, p, z)$ and substituting its untransformed value in the transformed equation (3.7), we arrive at an equation of the second kind on the contour Γ_4.

Spaces of functions and sequences with weight are introduced for a further study of the systems.

Let $C(\lambda)$ be the space of functions $f(z)$ which are continuous and vanish at infinity with the weight z^λ, and let $c(\sigma)$ be the space of sequences bounded and vanishing at infinity with weight p^σ. The norms in the mentioned spaces are given by the relationships

$$\| f(z) \|_{C(\lambda)} = \max_{z \in \Gamma_4} | f(z) z^\lambda | \qquad (3.12)$$

$$\| f \|_{C(\sigma)} = \sup_{|p| \leqslant \infty} | f_p p^\sigma |, \quad f = \{f_p\}$$

For the sake of simplicity, let us assume that the functions $f_k(r, \varphi)$ are infinitely differentiable with respect to each parameter in Ω_k. In this case the representation (2.6), in which $F_{kp}(\eta)$ decreases in p and η more rapidly than any power, can be obtained by continuing $f_k(r, \varphi)$ smoothly outside Ω_k.

Considering the system (3.6), (3.7) in the space

$$y(k, p, z) \in c(\sigma) \times C(\lambda) \quad (\sigma > m \geqslant 2,\ 0 < \lambda < 1,\ k = 1, 2) \qquad (3.13)$$

(the Cartesian product of the spaces $c(\sigma)$ by p and $C(\lambda)$ by z) and using uniform asymptotic estimates of the Bessel functions [8], which are also valid on the contour Γ_4 in the complex plane, we can prove the complete continuity of the operator in the right side of the transformed equations in this space. Now introducing $y(k, p, z)$ from the space (3.13) into (2.4), (2.5), and taking account of (2.1), it is easy to see that $q_k(\rho, \varphi)$ belongs to $L_\alpha(\Omega_k)$, $(\alpha > 1)$, i.e. the class of uniqueness. Hence, uniqueness and therefore, solvability of the system follow. The exact solution of the system (3.6), (3.7), which converges for all values of the parameters, can be written as a "Fredholm series" by using procedures of exterior analysis [4].

4. To construct an approximate solution of the problem, let us note that the infinite series in (3.7), (3.8) converge rapidly. Hence, they can be truncated by keeping N terms. The subsequent solution of the system of linear integral equations obtained by using approximate factorization can be reduced to solving an infinite linear algebraic system [1]. The matrix of the infinite system hence has only nonzero diagonal elements and column elements. Inverse matrices to those described are constructed easily [5].

Another approximate method consists of truncating the series (2.1) by the condition

$$f_{kl}(r) \equiv q_{kn}(r) \equiv 0, \quad k = 1, 2 \tag{4.1}$$

$$l = \pm(L+1),\ \pm(L+2),\ \ldots, \qquad n = \pm(N+1),\ \pm(N+2),\ \ldots$$

Consequently, the quantity of equations and unknowns in (3.7), (3.8) turns out to be $4N + 2$. The solution of this system can be obtained by using a Fredholm series or by constructing its approximate value by the method in [6].

5. Let us elucidate the subsequent scheme for solving the problem in the case of the vibration of two rigid plane stamps of radii and masses a_1, m_1 and a_2, m_2, respectively, on a laminar medium under the effect of forces $P_1 e^{-i\omega t}$ and $P_2 e^{-i\omega t}$.

We shall be interested in the vibration of the stamps relative to the static equilibrium position. The displacements of the described stamps have the following values in complex form (without the time factor)

$$f_1(r, \varphi) = c_1 + c_3 r e^{i\varphi} + c_4 r e^{-i\varphi}, \quad f_2(r, \varphi) = c_2 + c_5 r e^{i\varphi} + c_6 r e^{-i\varphi} \tag{5.1}$$

Here the constants c_1, c_2 characterize the vertical displacements of the stamps while the c_k $(k = 3, \ldots, 6)$ are linearly related to the angles of their rotation relative to the horizontal axes. All the constants listed should be determined. Using condition (4.1) for $N = L = 1$ and solving (3.7), (3.8) approximately, we find

$$q_n(r, \varphi) = \sum_{k=1}^{6} c_k q_{nk}(r, \varphi) \quad (n = 1, 2) \tag{5.2}$$

Let J_{k1} and J_{k2} be the moments of inertia of the stamps relative to the x- and y-axes on the undeformed surface. Then we represent the equations of motion of the stamps in the form

$$m_k \omega^2 c_k = \sum_{s=1}^{6} P_{ks} c_s - P_k, \quad k = 1, 2 \tag{5.3}$$

$$J_{kp}\omega^2 \sum_{m=1}^{2} \lambda_{pkm} c_{2k+m} = \sum_{s=1}^{6} M_{pks} c_s - M_{kp}, \quad k, p = 1, 2$$

Here the sums on the left are the angles of rotation of the stamps relative to the x- and y-axes $(p = 2)$, and in addition

$$P_{ks} = \iint_{\Omega_k} q_{ks}(r, \varphi)\, r\, dr\, d\varphi$$

$$M_{pks} = \iint_{\Omega_k} q_{ks}(r, \varphi)\, r^2 \cos\left[\varphi - (p-1)\frac{\pi}{2}\right] dr\, d\varphi$$

where M_{kp} is the moment of the force P_k relative to the x-axis ($p = 1$) or y-axis ($p = 2$). All the constants c_k are determined from the system (5.3).

Note 1. An investigation by the method from [1] turns out to be effective with the use of approximate factorization of the function $K[\mu_k(u)]$. In the presence of branch points A_k of degree $1/2$ on the real axis, where K vanishes (half-space) or becomes infinite, the approximating function should contain a factor or divisor of the form

$$\{(u^2 - A_k^2)[u^2 - (A_k + i\varepsilon)^2]\}^{1/2}, \quad 0 < \varepsilon \ll 1$$

This method permits making the integral over the slit $-A_k$, $-A_k - i\varepsilon$ small for small ε, which is naturally obtained in (3.7), (3.8) for deformation of the contours Γ_k in the lower half-plane. If the value of K at the branch points differs from zero and infinity, then such branch points are considered nonsingular.

Note 2. The method elucidated is developed for the case of any finite number of stamps, where the case of two is presented here for the sake of brevity. The branch points of the function $K(u)$ occur in the presence of an elastic half-space on which the elastic layers rest. The method is also applicable in the case of an absolutely rigid half-space or in its absence (packet of layers). In this case $K(u)$ is a meromorphic function.

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